Discussion of "Subjective Expectations and Equilibrium Yield Curves"

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U.S. Yields

Figure 1: **U.S. 10- and 1-year Treasury bond yields**

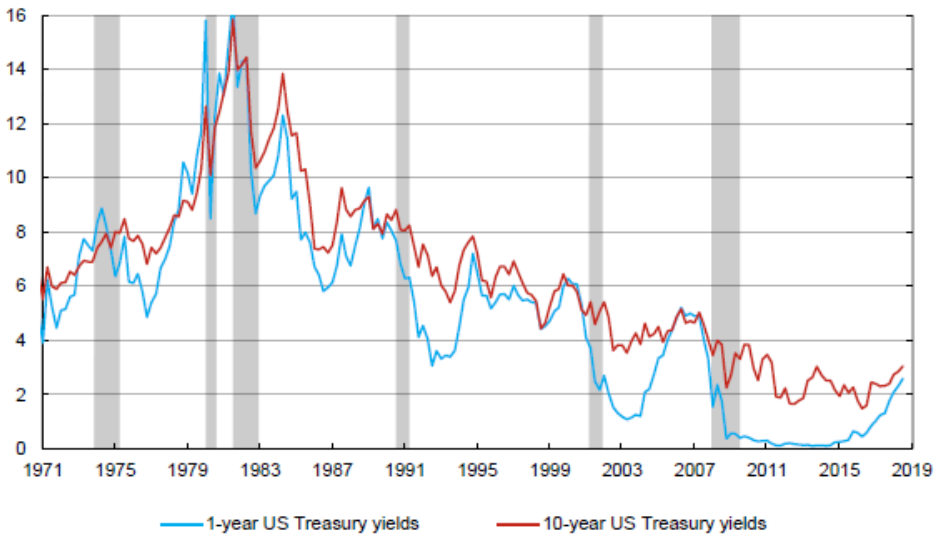
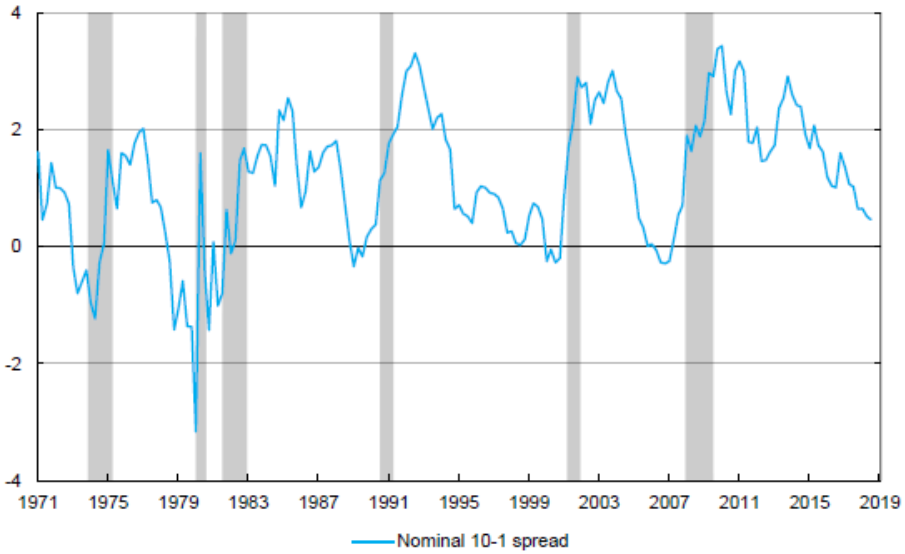


Figure 1: **US Treasury yield spread (10- minus 1-year)**



Question

- To what extent can an asset pricing model with non-rational expectations and learning about growth and inflation explain the dynamics of the yield curve?

- **Key Facts**:

1. Low-frequency trend in yields: run-up post WWII until the early 1980s and long secular decline afterwards

2. Large cyclical variation around the trend in both short-yields and yield spreads

3. Positive trend and increased persistence in yield spreads

Model - Key Ingredients

- Representative agent, endowment economy with CRRA utility
- Distorted likelihood updating

$$p\left(x_t|I_t\right) \propto p\left(y_t|x_t\right)^{1+\theta} p\left(x_t|I_{t-1}\right)$$

  where θ governs deviations from rationality and Bayesian updating

- Learning about trend and gap components in endowment growth and inflation
- Exponentially affine pricing kernel for tractable bond pricing

Model - State Space System

- **Key Equations:**

$$\Delta g_t = \Delta g_t^* + Gap_t^g$$
$$\pi_t = \pi_t^* + Gap_t^\pi$$

- Dynamics of trend components:

$$\Delta g_t^* = \mu_{g,t} + \sigma_g \varepsilon_{g,t}^*$$
$$\pi_t^* = \mu_{\pi,t} + \sigma_\pi \varepsilon_{\pi,t}^*$$
$$\mu_{i,t+1} = \mu_{i,t} + \sigma_i^\mu \varepsilon_{i,t+1}^\mu \quad \text{for } i = g, \pi$$

- Dynamics of gap components:

$$Gap_t^i = x_{i,t} + \sigma_i^{gap} \varepsilon_{i,t}^{gap}$$
$$x_{i,t+1} = \rho_i x_{i,t} + \sigma_i^x \varepsilon_{i,t+1}^x$$

Comment 1: Simplify/Clarify the Framework

- What is the purpose of having so many error terms and are their volatilities even separately identified?
- Why not use something like the following system, with Δg_t as an example:

$$\begin{aligned}
\Delta g_t &= \mu_{g,t} + x_{g,t} \\
\mu_{g,t+1} &= \mu_{g,t} + \sigma_g^{\mu} \varepsilon_{g,t+1}^{\mu} \\
x_{g,t+1} &= \rho_g x_{g,t} + \sigma_g^{x} \varepsilon_g^{x,t+1}
\end{aligned}$$

- You could add a purely transitory shock to the first equation if desired, but it's not necessary

Comment 1: Simplify/Clarify the Framework

- The definition of the trend and gap components is confusing and non-standard:

$$\Delta g^*_{t+1} = \frac{C_{t+1} - C_t}{GDP_t}$$

$$\pi^*_t = \frac{P^{core}_{t+1} - P^{core}_t}{P_t}$$

- Why not follow the consumption-based asset pricing literature and use growth in real consumption per capita as the measurable? Or simply use growth in real GDP per capita?
- Similarly, why focus on core inflation? I was also confused because at various points you mention both PCE inflation and GDP deflator inflation.

Comment 2: Justification of the Learning Rule

- What does the proposed learning rule buy us relative to more standard specifications?
- The evidence on over and under-reaction is inconsistent across macro variables and differs at the individual vs the consensus level
- How does the model estimated under rational Bayesian learning perform? This would be a natural benchmark to help isolate the role of non-rational expectations
- See Crump et al. (2022): simple multivariate setup similar to this paper which can rationalize the observed time series movements in nominal yields and survey expectations on inflation, GDP, and interest rates

Comment 3: Estimation of the Model

- I do not understand what the information set of the agents is
- My takeaway is that agents can separately observe the trend and gap components of output growth and inflation with measurement error, e.g. Δg_t^* and Gap_t^g for growth
- If so, why? This is the most important part of the learning problem, disentangling trend from cycle!
- See e.g. Farmer et al. (2024), which can match the dynamics of yields and under-reaction of forecasts with rational Bayesian learning of a trend/cycle decomposition under parameter uncertainty

Comment 3: Estimation of the Model

- What data were used for estimation? Were forecasts explicitly incorporated? Were yields explicitly incorporated?

- If inflation, output growth, forecast, and yield curve data are all explicitly used in estimation I do not see why you could not jointly estimate all parameters including preference parameters and latent state autocorrelations

- No standard errors for parameter and state estimates were reported. Is there large uncertainty around these objects or are they precisely identified?

Comment 3: Estimation of the Model

- One of the main stories you tell is about the changing correlation between the cyclical components of real GDP and inflation
- Why not explicitly incorporate this into your model and test for its significance?
- Allow for a break in the correlation between $x_{g,t}$ and $x_{\pi,t}$ and test for a significant change in this parameter. This could be through a change in VAR dynamics or in the correlation between shocks to each series
- Explicit comparison of fitted moments pre and post-1990 between your model and the data would help bolster you conclusion that you match the shift in the dynamics of the yield spread

Comment 4: Time-Varying Risk Premia

- What is the role of time-varying risk premia quantitatively?
- Your model rules out a role for time-varying risk premia but given the low frequency fluctuations in output growth and inflation you allow for, long-run risks could play a significant role
- It is potentially important to have a version of your model with recursive preferences and stochastic volatility if you want to more directly make contact with the asset pricing literature
- How much of yield dynamics are expectations driven vs. risk premia driven?

Comment 5: Forecast Anomalies

- There are several other notable facts about survey forecasts which are anomalous beyond under-reaction, such as persistent bias and autocorrelation in forecast errors
- Can your model touch on these issues? These anomalies can serve as additional validation tests
- The degree of under-reaction appears to be dramatically over estimated given my understanding of empirical estimates in the literature

Conclusion

- This is an interesting paper that in my opinion, pushes in the right direction
- Developing theories which can match the dynamics of subjective expectations and using those subjective expectations to discipline our frameworks makes a lot of sense
- Some small changes could push the project even further
- There is a need to place the paper more definitely in a specific literature